Notes on Factoring by *GCF* - Page I Name_________________________

Perhaps, the process of factoring by removing the greatest common factor can be best stated as the ***reverse distributive property***. In the distributive property, one is multiplying a certain factor to all of the terms. In factoring by *GCF*, one is dividing all of the terms by the *GCF*.

Consider this expression which utilizes the distributive property: $5x^2(4x^4+3)$.

Visually, this is the distributive process: $5x^2(4x^4+3)$.

To simplify using the distributive property, one multiplies $5x^2$ times $4x^4$, and then one multiplies $5x^2$ times 3.

$$5x^2 \bullet 4x^4 = 20x^6$$

$$5x^2 \bullet 3 = 15x^2$$

After simplifying using the distributive property, you get $\boxed{20x^6+15x^2}$.

This section will now demonstrate how to factor by removing the *GCF*.

Let's now take your ***answer*** to the problem above: $20x^6+15x^2$.

Using what was learned in the last lesson, the *GCF* of $20x^6$ and $15x^2$ is $\boxed{5x^2}$.
Recall - this is because the greatest common factor of 20 and 15 is 5, and because the *GCF* of like variable quantities is always the lowest exponent.

Now, **divide** each term in the original expression by the *GCF* ($5x^2$). Divide $20x^6$ by $5x^2$, and divide $15x^2$ by $5x^2$.

$$20x^6 \div 5x^2 = 4x^4$$

$$15x^2 \div 5x^2 = 3$$

Therefore, after dividing by the *GCF*, the expression is $4x^4+3$.

To complete this **reverse distributive process**, write the *GCF* in front of a set of parentheses. Inside of the parentheses, place the expression that is left after dividing by the *GCF*.

$$= \underbrace{5x^2}_{GCF} \; (\underbrace{4x^4 \quad + \quad 3}_{what's\ left\ after\ dividing})$$

So, after factoring by removing the *GCF*, the answer is $\boxed{5x^2(4x^4+3)}$. Note how this is the original question before distributing at the very top of the page.

Factor the greatest common factor: $8y^5-12y^3+4y$.

The *GCF* is of the three terms is $4y$, because the GCF of 8, 12, and 4 is 4, and the *GCF* of y^5, y^3, and y is y. So, the *GCF* ($4y$) will be placed in front of the parentheses, and all of the terms in the expression will be divided by $4y$.

$$
\begin{array}{c}
8y^5 - 12y^3 + 4y \\
\downarrow \div 4y \quad \downarrow \div 4y \quad \downarrow \div 4y \\
= \underbrace{4y}_{GCF} \; \underbrace{(2y^4 - 3y^2 + 1)}_{\text{what's left after dividing}}
\end{array}
$$

Therefore, the answer is $\boxed{4y(2y^4 - 3y^2 + 1)}$.

Generating the last term in this expression is where many students make a mistake. In order to get "+1", one has to divide *4y* by *4y*. Some students would think this is zero, and they would not write anything. However, it's important to see that $4y \div 4y = \boxed{1}$.

Factor the greatest common factor: $14z^8 + 24z^7 - 30z^3$.

First, the *GCF* of all three terms is $2z^3$. Now, divide each of the terms by $2z^3$.

$$
\begin{array}{c}
14z^8 + 24z^7 - 30z^3 \\
\downarrow \div 2z^3 \quad \downarrow \div 2z^3 \quad \downarrow \div 2z^3 \\
= \underbrace{2z^3}_{GCF} \; \underbrace{(7z^5 + 12z^4 - 15)}_{\text{what's left after dividing}}
\end{array}
$$

The answer is $\boxed{2z^3(7z^5 + 12z^4 - 15)}$.

Factor the greatest common factor: $16c^7 - 6c^3$.

The *GCF* is $2c^3$. Now, you complete the problem below:

$$
\begin{array}{c}
16c^7 - 6c^3 \\
\downarrow \div \quad \downarrow \div \\
\underbrace{\_\_\_\_\_\_}_{GCF} \; \underbrace{(\quad - \quad)}_{\text{what's left after dividing}}
\end{array}
$$

For Questions 1-2, factor the greatest common factor.

1. $25d^5 + 45d^4$

2. $9k^4 + 12k^3 - 6k$

Notes on Factoring by *GCF* - Page II Name__________________________

Factor the greatest common factor: $28a^3b^2 - 36a^2 - 17b^5$.

Note that the *GCF* of the coefficients (28, -36, and -17) is 1. Also, note that the terms do not all share any common variables.

Obviously, it makes little sense to write $1(28a^3b^2 - 36a^2 - 17b^5)$.

When one is only factoring out the greatest common factor, and **the *GCF* is 1**, he/she should write that the expression is **PRIME** .

Homework on Factoring by Greatest Common Factor

Factor the greatest common factor out of the polynomial. If the *GCF* is 1, write *PRIME.*

1. $8x^2 + 10x$
2. $12y - 16$
3. $-15d^5 + 45d^3$
4. $13a + 20b$
5. $c^3 + c^2 - c$
6. $6n^2 - 30n + 42$
7. $-7m^2 - 10m + 17$
8. $18p^3 - 63p^2 - 9p$
9. $18x^2 - 50y^2$
10. $100z^9 + 50z^6 - 75z^5$
11. $36rs^2 - 108r^2s^3$
12. $36k - 30$
13. $a^7b - a^{10}$
14. $2c^5d^4 - 3c^4 + 4c^3$
15. $3g^8 + 3g^7$
16. $18x^5 - 48x^4 + 56x^3 - 86x$
17. $23y^{10} - 46y^7 + 68y^2 + 10y$

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1. $2x(4x+5)$
2. $4(3y-4)$
3. $15d^3(-d^2+3)$ or $-15d^3(d^2-3)$
4. PRIME
5. $c(c^2+c-1)$
6. $6(n^2-5n+7)$
7. PRIME
8. $9p(2p^2-7p-1)$
9. $2(9x^2-25y^2)$
10. $25z^5(4z^4+2z-3)$
11. $36rs^2(1-3rs)$
12. $6(6k-5)$
13. $a^7(b-a^3)$
14. $c^3(2c^2d^4-3c+4)$
15. $3g^7(g+1)$
16. $2x(9x^4-24x^3+28x^2-43)$
17. $y(23y^9-46y^6+68y+10)$